Housing and Consumption: Focus on the Elasticity of Intra-temporal Substitution

송인호 (In-Ho Song)*

< Abstract >

Studies to define the relationship between housing and macroeconomics along with financial crisis have never been more active than in the recent years. One of the most popular studies is to analyze the mechanism of house prices in relation to the financial market and macro-economy. In particular, the elasticity of intra-temporal substitution (EIS) between housing and consumption, together with the inter-temporal elasticity of substitution (IES), is essential in identifying the effect of house prices on macroeconomic variables. Preceding studies so far have their models mainly focused on the EIS between durable and non-durable goods consumption, but paid less attention to the EIS between housing and consumption. This paper is designed to estimate the EIS between the two series through the Bayesian estimation method by using Korean macro-data, which is a new approach. First, I develop a constant elasticity of substitution (CES) utility function and define consumers' utility maximization problems with housing as collateral. Then I derive an empirical estimation model by linearizing the first-order conditions and Euler equation. Finally, the result shows the estimate of the EIS, 0.39, implying that housing and consumption are complements. This finding helps construct a macroeconomic housing model which better fits the data, generating the co-movement between housing and consumption. Accordingly, macroeconomic models with complementarity between housing and consumption provide the full-scale of studies as to the role of houses in macroeconomic conditions. To the extent to which housing and consumption are complements, the two series tend to move together. For robustness check, the Canonical Cointegration Regression (CCR) estimation is conducted. The result is also consistent with complementarity between housing and consumption.

Keywords: Bayesian Estimation, The elasticity of intra-temporal substitution (EIS), The inter-temporal elasticity of substitution (IES), Co-integration, CCR

* 한국개발연구원, 거시금융정책연구부 연구위원, 경제박사 (inhosong@kdi.re.kr or inhosong@gmail.com)
I. Introduction

It was only from 2008 that housing was considered within the mainstream of macroeconomics and regarded as one of key drivers affecting the real economy. Hit by the financial crisis resulting from the US sub-prime mortgage crisis in late 2007 and by the collapse of Lehman Brothers in September 2008, the global economy lapsed into the longest recession of 1.6 years (December 2007 to June 2009) since the Great Depression in the 1930s. Reinhart and Rogoff (2009) analyze that the recent economic setback shows a longer downfall on average and it tends to have a much stronger negative impact on the national economy, compared to when the downturn is unrelated to the housing market. After weathering the crisis, several studies have been intensely conducted to explain the relationship between the housing market and economic fluctuations. Studies to define the relationship between housing and macroeconomics along with financial crisis have never been more active than in the recent years.

Housing is newly introduced into macroeconomic models, which is an emerging trend in a sense that housing is frequently used in a variety of macroeconomic housing research\(^1\). In particular, the Dynamic Stochastic General Equilibrium (DSGE) models are most popularly used to understand the housing price mechanism in connection with macroeconomy. Hence, the partial market perspective on the housing research recently started to shift toward a general market approach by focusing on the linkages between the housing market and macroeconomic variables. As a new trend of macroeconomic housing approach, general equilibrium model settings are to directly treat housing and consumption as an objective preference function in order to account for the comovement between the two series. However, most macroeconomic housing models are constructed with a log separable utility function over the two series, by ignoring the unique characteristics of housing. In other words, under such model settings, the interactions between housing and consumption are paid less attention so that the full scale of interaction effects between the elasticity of

---

1) Iacoviello(2005) and Iacoviello and Neri(2010) recently introduced housing into DSGE models to account for the economic fluctuations.
intra-temporal substitution (EIS) and inter-temporal elasticity of substitution (IES) on macroeconomic fluctuations may not be fully studied. Since the magnitude of the effect in policy changes depends on the degree to which the EIS interacts with the IES\(^2\), finding the estimate of the EIS and the IES is an important agenda in macroeconomics. In particular, the EIS plays an essential role in determining the relationship between housing and consumption. If the EIS is estimated to be much larger than 1, housing and consumption would serve as stronger substitutes to each other. Or, if this is estimated to be much less than 1 and close to 0, both goods would serve as stronger complements to each other.

First, this paper is to define the relationship between housing and consumption by estimating the EIS, and then to help construct macroeconomic housing model settings fitting the macro-data. To that end, I used the Bayesian estimation method and the Metropolis-Hastings algorithm (An and Schorfheide (2007)). Given that houses serve as collateral, the LTV(loan-to-value) is introduced to the empirical model. I abstract the first-order conditions from the micro-foundation consumer optimization problem and then linearized these conditions. The macro data come from the Korea’s Cheonsei price index, interest rates, non-durable goods consumption and the CPI, which are used to construct housing service and non-durable goods consumption. According to the results, the EIS between housing and consumption is estimated at 0.39, implying strong complementary between the two series. Moreover, in order to ensure the robustness of its result, using the same first order conditions and the Euler equation in the Bayesian estimation method, I also conducted the Canonical Cointegration Regression (CCR) approach, and the result is found to be consistent with findings by the Bayesian estimation result\(^3\). This confirms that housing

---

2) As Ogaki et al. (1998) argue, the EIS in macroeconomic models plays an important role in explaining characteristics of both durable and non-durable goods consumption, as that of IES does.

3) Lee (2003) also estimated the elasticity of intra-temporal substitution using other macro-data released by the Bank of Korea through co-integrating methods. Differently from Lee (2003), housing services, which I use, are not included in non-durable consumption in the Korean national account data. In my paper, the estimation through the Bayesian method turned out 0.39, implying that this study can be considered in the same context with the result of latest data-based cointegration, as an empirical model including the mortgage loan effect. In addition, I used a Dynamic Ordinary Least Square (DOLS) method to estimate the intra-temporal substitution. Other cointegration methods I used are Canonical Cointegrating Regression (CCR), and Fully
and consumption are complementary goods to each other. In the case of a strong complementary relationship, a rise in housing prices would provide a strong comovement between housing and macroeconomic variables. The complementarity causes macroeconomic models to better generate the comovement between housing and consumption. This complementarity also helps to better explain the equity premium puzzle, which is taken into account for the asset price theory by Piazzesi, Schneider, and Tuzel (2007). In addition, the result of this paper also supports Hong(2010)'s assumption that the EIS should be smaller than the IES in explaining the puzzle.

The paper consists of seven chapters including the introduction (Chapter I) and conclusion (Chapter VII). Chapter II and Chapter III show the Korean housing characteristics and the literature review, respectively. Considering the characteristics of Korean housing, the empirical model to estimate the EIS between housing services and consumption is constructed in Chapter IV. And Chapter IV presents the empirical model for the Bayesian estimation method and its results. Chapter V and Chapter VI show the robustness check and implications of results, respectively.

II. Characteristics of Korean Housing

The housing market and the real economy tend to move together. In fact, housing, being treated as a key axis of the economic fluctuation, has led to the emergence of a new issue as to the relationship between housing and economic fluctuations.

---

Modified Ordinary Least Squares (PMOLS).

4) The wealth effect of assets (Song, 2012, Yoon, 2012) is used to give an explanation for the comovement.
[Figure 1] shows the comovements between real house prices and GDP, which can be seen in line with the complementary relationship between the two suggested in this study. Since consumption is a major factor of GDP, the comovement between the real housing price and GDP implicitly implies the complementarity goods between housing and consumption. In order to study the linkages between Korean macro-data and housing, the characteristics of housing in Korean data may need to be understood.

Korean housing has specific characteristics in several aspects: (1) [Figure 2] displays Korean average LTV ratio. The ratio has recently hovered around 50% on average, comparatively lower than other OECD countries. Although many households use mortgage, it is not the case in all households. The average rate in [Figure 2] is based on mortgage users.

<table>
<thead>
<tr>
<th>Date</th>
<th>Country</th>
<th>Metropolitan</th>
<th>Rural</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec. 2008</td>
<td>46.4</td>
<td>44.4</td>
<td>52.9</td>
</tr>
<tr>
<td>Dec. 2009</td>
<td>46.4</td>
<td>44.6</td>
<td>52.3</td>
</tr>
<tr>
<td>Dec. 2010</td>
<td>47.3</td>
<td>46.1</td>
<td>51.1</td>
</tr>
<tr>
<td>Dec. 2011</td>
<td>47.4</td>
<td>46.8</td>
<td>48.4</td>
</tr>
<tr>
<td>Mar. 2012</td>
<td>47.8</td>
<td>47.2</td>
<td>48.7</td>
</tr>
</tbody>
</table>

Source: individual banks (9banks). Metropolitan area includes Seoul, Kyunggido, and Incheon. Financial supervision, Financial statistic information system.

[Figure 2] Korea’s LTV Ratio (%)
Korean housing services\(^5\) may be represented by its unique rental system, called Cheonsei\(^6\). Unlike other durable goods, housing is closely related to the mortgage loan system, thereby tightly connected to the bank’s credit channel. In general, houses are highly leveraged. House prices can be seen as the sum of land prices and building prices, and these prices are much larger than GDP. More importantly, houses, together with the land it stands on, can play dual roles: housing services and investment assets. In fact, houses are a major household asset. According to [Table 1], the asset structure of Korean households consists of mainly real estate property (74\%) and financial assets (23\%).

*Table 1* Asset Structure of Korean Households (as of 2011)

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Household Asset (KRW 10,000)</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Financial Asset</td>
</tr>
<tr>
<td>2010</td>
<td>27,684</td>
<td>5,886</td>
</tr>
<tr>
<td>2011</td>
<td>29,765</td>
<td>6,903</td>
</tr>
</tbody>
</table>

Source: KOSIS, Statistics Korea

III. Literature Review

Studies on the estimation of the EIS between housing and consumption have been deficient since 2003, so that most macroeconomic housing models construct the separability preferences over the two series. Using Korean micro-data, Lee (2003) estimated the EIS around 0.2 through a co-integration method. In particular, Lee’s data\(^7\) come from water usages and electronic services, which are used to derive housing services\(^8\). In addition, his

---

5) In the US, housing service prices are imputed rental prices calculated in the NPA data, calculating the case of leasing self-owned houses.

6) The data on Cheonsei price are treated as important data, as is house purchase price. In Cheonsei, the tenant is required to deposit about 50\% of house prices at rental contract to house owners. At expiry date of the contract, the deposit is fully refundable with zero interest to the tenant.

7) Lee’s Korean data ranging from 1986 Q1 to 2003 Q1 provide evidence for complementarity. I use the data for consumption and housing services ranging from the first quarter of 1991 to the first quarter of 2011.
empirical model is not related to housing mortgage, so that housing collateral may not be integrated in the estimation. Hence, his result does not provide the estimate of both the EIS and IES, associated with housing collateral effect.

Consistent with Korean micro-data, US micro data also provide evidence for the complementarity relationship between housing and consumption, hence support for non-separable utility functions over these two time series. Stokey (2007) used the CEX data and found the EIS estimate of 0.23. In addition, Hanushek and Quigley (1980) used HADE (Housing Allowance Demand Experiment) data, presenting complementarity. Moreover, Flavin and Nakagawa (2004) investigated the relationship between these two series using PSID data, and estimated the EIS at 0.13 through GMM methods. Li et al. (2009) also focused on the EIS in PSID data and estimated it at 0.33 using the different method such as simulated moments (MSM).

However, the most macro-model based housing literature ignores the micro-data evidence by constructing separability preferences over housing and consumption. Since separability is associated with independency between the two series, it is hard to analyze the interaction effects between the EIS and the IES on macroeconomic variables.

IV. Bayesian Estimation

1. Representative Consumer

The micro-foundation consumer optimization problem is adopted from Song (2012) in order to construct an empirical model for the Bayesian estimation framework. Since the main

8) The data in his paper is not related to macro-data. It is necessary that the complementarity between housing and consumption be richly tested by macro-data in order to help to construct a macro model.

9) These findings are also consistent with the results in Siegel (2004).

10) Song (2012) conducted a cointegrating test for the EIS based on setting up the representative consumer problem. Song (2011) also used the representative consumer problem, but there were no houses as collateral in budget constraints.
focus of the paper is to estimate the EIS between housing and consumption, it would be appropriate to assume that the economy be for a partial equilibrium model\(^{11}\). And, it is assumed that the preference for housing, \(D_t\), and non-durable goods consumption, \(C_t\), are always connected to the CES utility function. The representative consumer obtains utility by consuming housing services and non-durable consumer goods.\(^{12}\) \(\beta\) represents the subjective discount rate of the representative consumer and is assumed to be smaller than 1. In the maximization of the utility function for the representative consumer, the interest rate, \(R\), is associated with mortgage, \(b_t\), which implies the amount of money that the consumer can borrow by using his/her house as collateral. At any point in time, \(t\), the utility maximization problem of the representative consumer is defined as below.

\[
\max \ E_0 \sum_{t=0}^{\infty} \ (\beta)^t \ U(C_t, D_t),
\]

such that

\[
C_t + h_t(D_t - (1 - \delta)D_{t-1}) + Rb_{t-1} = T_tD_{t-1}^\nu + b_t
\]

Equation (2) denotes the inter-temporal budget constraint. Housing prices, \(h_t\), are normalized at 1 and the transaction of housing is assumed to take place at every period. The left side of Equation (2) denotes the total spending, whereas the right side the total income. I assume that output is produced by means of housing as a unique input element.

\(^{11}\) For plentiful analysis about the housing and macroeconomics, the economy is assume to be for general equilibrium model including firms, government, and households. In this economy, however, there is a representative consumer.

\(^{12}\) In the past, durable and non-durable goods consumption were considered as composition elements of consumption, but this trend has changed after housing was included in the macroeconomic model. Recent literatures have included housing service consumption—which has the nature of durable goods—and non-durable goods consumption as composition elements when setting utility functions of consumption. Some argue that in the context of representing overall consumption, housing and non-housing consumption should be used as composition elements. Meanwhile, others claim that the inherent meaning of non-housing consumption remains mixed with durable and non-durable goods consumption, distorting the accurate elasticity of consumption utility function. This study avoids discussions regarding the sum of consumption, as the consumption in this study operates only in the partial equilibrium model.
\[ Y_t = T_tD_{t-1}^v \], depending on a constant return to scale with technology, \( T \). In this function, the technology is expressed as \( T = \exp(Z_t) \), and \( Z_t \) is a stochastic shock to technology. This stochastic shock process is assumed to be achieved through \( Z_t = \rho Z_{t-1} + \omega_t \), where \( E(\omega_T) = 0, E(\omega_t, \omega_T) = \sigma_T^2 \).

Also, the constraint on mortgage loan is expressed as below.

\[ b_t \leq lvv \cdot D_t \]  

(3)

\( lv \) is assumed to be not greater than 1, meaning \( 0 \leq lv \leq 1 \). Hence, the consumer is not allowed to receive mortgage greater than the house value. In solving the optimization problem, the mortgage loan is assumed to be always calculated according to the LTV ratio, \( lv \). Indeed, Korea’s LTV ratio hovers around average 50%, relatively lower than other OECD countries.

Most importantly, in order to estimate the EIS, the utility function is defined as below.

\[ U(C_t, D_t) = \frac{1}{1-1/\sigma} \left[ \left( \frac{\varepsilon-1}{\varepsilon} C_t^\varepsilon + (jD_t)^{\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right]^{1-1/\sigma} \]  

(4)

In Equation (4), \( j \) is assumed to be a constant ratio element which makes houses as housing services. The important parameter in the utility function of representative consumer is \( \varepsilon \) and \( \sigma \): the former represents the EIS between housing services and non-durable goods consumption, and the latter represents the IES between the two. As the former gets closer to infinity, housing services and non-durable goods consumption become more perfect substitutes. As the former gets closer to 0, they become more perfect complements. The sum of the expected future discounted utility is expressed in time, 0, as below.

Finally, this utility maximization problem of representative consumer can be denoted as below.
\[ \Lambda = E_t \sum_{t=0}^{\infty} (\beta)^t \frac{1}{1-1/\sigma} \left[ \left\{ C_t^{\frac{\sigma-1}{\sigma}} + j(D_t)^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}} \right]^{1-1/\sigma} \]

\[ + \lambda_t (Y_t + b_t - C_t - (D_t - D_{t-1}) - Rb_{t-1}) + \lambda_{2t} (ltv \cdot H_t - b_t) \]  
(5)

Based on Equation (5), the first-order conditions on non-durable goods and housing service consumption and the mortgage loan can be obtained through the following.

\[ U_{C_t} = \lambda_t = \left\{ \frac{\frac{\sigma-1}{\sigma}}{C_t^{\frac{\sigma}{\sigma-1}} + (D_t)^{\frac{\sigma-1}{\sigma}}} \right\}^{\frac{1}{\sigma-1}} \left( 1 - \frac{\sigma}{\sigma} \right) \frac{1}{C_t^{\frac{\sigma}{\sigma-1}}} \]  
(6)

\[ U_{D_t} = \left\{ \frac{\frac{\sigma-1}{\sigma}}{C_t^{\frac{\sigma}{\sigma-1}} + (D_t)^{\frac{\sigma-1}{\sigma}}} \right\}^{\frac{1}{\sigma-1}} \left( 1 - \frac{\sigma}{\sigma} \right) \frac{1}{(j_{t}D_{t})^{\frac{\sigma}{\sigma-1}}} \]  
(7)

\[ = U_{C_t} + U_{C_{t+1}} \beta (1 - T_{t+1} D_t) - \lambda_{2t} \cdot ltv \]

\[ U_{C_t} = U_{C_{t+1}} \beta + \lambda_{2t} \]  
(8)

Moreover, I combine the first-order conditions for the representative consumer utility maximization problem. Equation (6) and Equation (8) produce the Euler equation.

\[ \frac{U_{D_t}}{U_{C_t}} = \frac{h_t}{p_t} (1 - m) + \frac{U_{C_{t+1}}}{U_{C_t}} h_{t+1} \beta (1 + ltv) = \left( \frac{D_t}{C_t} \right)^{\frac{1}{\sigma}} j^{\frac{1}{\sigma}} \]  
(9)

This equation can be interpreted as below in the context of relative consumption.

\[ \log \left( \frac{C_t}{D_t} \right) = \log(j) + \epsilon \log \left( \frac{h_t}{p_t} (1 - ltv) + \frac{U_{C_{t+1}}}{U_{C_t}} \frac{h_{t+1}}{p_t} \beta (1 + ltv) \right) \]  
(10)

Meanwhile, I assume that the growth rate of the marginal utility consumption and the real housing appreciation rate are identical to the relative price of housing to consumption. Equation (10) can be re-expressed as below.

\[ \log \left( \frac{C_t}{D_t} \right) = \log(j) + \epsilon \log \frac{h_t}{p_t} + \epsilon \log((1 - ltv) + \beta (1 + ltv)) \]  
(11)

The parameter, j, is assumed as a constant and draws housing service, and \( h_t \), as the parameter of the LTV, is as well assumed as a constant rate. The subjective discount rate is assumed as
the parameter of a constant, which makes the function of $\log(j) + \epsilon \log((1 - ltv) + \beta(1 + ltv))$ constant. I assume it to be $f(j)$, so as to give the equation as below.

$$\log\left(\frac{D_t}{H_t}\right) = f(j) + \epsilon \log\frac{h_t}{p_t}$$  \hspace{1cm} (12)

2. Bayesian Approach

The Bayesian approach has been recently adopted in estimating the parameters for macroeconomic model settings. Provided that a number of endogenous variables are observed, some or all parameters can be estimated by Dynare\textsuperscript{13} through both maximum likelihood and Bayesian techniques. In Bayesian approach, the unknown parameters that I focus on are $\omega, \sigma, \beta, j, v, \rho, \omega_A$, which I estimate from the observed data, housing services and non-durable consumption. The first approach to this Bayesian estimation is to build around a likelihood function, which can be abstracted from the above Equation (1) through Equation (8). Most techniques and algorithms have been developed in the literature, including An and Schorfheide (2007). Specifically, the technique of Markov Chain Monte Carlo (MCMC) methods is applied to this Bayesian approach. The strong point of the Bayesian estimation method is to well fit the time series within the model structures, particularly, a DSGE model. The model generates the likelihood function, which is the groundwork of estimation. Additionally, prior distribution information is used for parameter estimation, which provides more accurate identification. It is well known that there are identification problems due to lack of information in structural parameter estimation methods. I borrow the MCMC technique, Metropolis-Hastings algorithm, and Random-Walk metropolis (RWA) algorithm from An and Schorfheide (2007). Data to be used for estimation are non-durable goods consumption taken from the national account released by the Bank of Korea and Cheonse price index released by Kookmin Bank. To set up actual real value data, CPI is used. The Cheonse price can be seen as a price index that represents Korea’s housing services.

\textsuperscript{13} I use “Dynare”, Reference Manual, version 4.2.5.
3. Data

Unlike other OECD nations, Korea’s Cheonsei system requires tenants to pay approximately 50% of the house price as a deposit in advance for a two-year lease, and at the maturity of contract period, the deposit is given back without interest to tenants. In other words, the actual implicit earning of house owners is equal to the amount obtained by multiplying the initial lump sum deposit by the interest rate over the two-year term. As a way to abstract housing services, Cheonsei prices are multiplied by the interest rate. All data range from 1991 Q1 to 2011 Q1. Basic data is displayed in [Figure 3]. Non-durable goods consumption may illustrate a stochastic trend, which in general is difference stationary. The Augmented Dickey Fuller test for difference stationary is conducted as in [Table 2].
<table>
<thead>
<tr>
<th>Time series</th>
<th>ADF Test</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \log(C_t / H_t) ) has a unit root</td>
<td>-1.37</td>
<td></td>
</tr>
<tr>
<td>( \log(q_t / p_t) ) has a unit root</td>
<td>-0.91</td>
<td></td>
</tr>
<tr>
<td>First difference of time series</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D[\log(C_t / H_t)] ) has a unit root</td>
<td>-10.67***</td>
<td></td>
</tr>
<tr>
<td>( D[\log(q_t / p_t)] ) has a unit root</td>
<td>-8.91***</td>
<td></td>
</tr>
</tbody>
</table>

According to the Augmented Dickey Fuller test results, the null hypothesis that \( \log(C_t / H_t) \) has a unit root cannot be rejected. However, the unit root test for \( D[\log(C_t / H_t)] \), after first-differencing, shows the null hypothesis can be rejected. Accordingly, the null hypothesis that \( D[\log(q_t / p_t)] \) has a unit root is being rejected, meaning that both the relative price of housing to consumption, and the relative consumption are difference stationary. These unit root tests indicate that the cointegrating estimation approach is valid, which can be applied to the CCR estimation in Section V.

### 4. Parameterization and Estimation

To estimate parameters in the structure of the Bayesian method, I focus on combining the first-order conditions for optimization, Equation (6)–(8), to the data, so that the elasticity of the intra-temporal substitution, \( e \), may be estimated. Since the EIS is a link parameter directly between housing service and non-durable goods consumption, its time-series are directly used for parameter estimation. The EIS is also connected with that of the IES and also directly related to \( \beta \), the subjected discount rate of representative consumer, and \( j \) which transform the houses into housing service, and therefore these parameters are for estimation targets.

Then, based on the first moment value of the data, remaining parameters are parameterized. The LTV ratios are set to be 50% according to the mortgage data from Financial Supervision Agency as in [Figure 4]. Korean LTV across 2005 through 2012 and mortgage LTV constitution is a basis for parametrization of the Korean LTV. However, in the future research for rigorous analysis, the LTV ratio may be weighted with respect to different level mortgage constitution,
since in some cases, some households do not use mortgage.

<table>
<thead>
<tr>
<th>Year</th>
<th>LTV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005 Q2</td>
<td>47.0</td>
</tr>
<tr>
<td>2006 Q2</td>
<td>49.4</td>
</tr>
<tr>
<td>2007 Q2</td>
<td>51.0</td>
</tr>
<tr>
<td>2008 Q2</td>
<td>48.6</td>
</tr>
<tr>
<td>2009 Q2</td>
<td>47.7</td>
</tr>
<tr>
<td>2010 Q2</td>
<td>48.2</td>
</tr>
<tr>
<td>2011 Q2</td>
<td>50.6</td>
</tr>
<tr>
<td>2012 Q1</td>
<td>51.1</td>
</tr>
</tbody>
</table>

Source: Left Table: Financial supervision, and Financial statistic information system ; Right Table: individual banks (%banks).

(Figure 4) Korea*’ s LTV Ratio (%)

And the depreciation rate of houses is made non-existent as many other macroeconomic models have assumed so far. The proportion of the gross domestic production by house is set to 0.05. Parametrization details are described in [Table 3].

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>Housing Depreciation Rates</td>
<td>0.0 %</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Proportion of houses to the gross domestic production</td>
<td>0.05%</td>
</tr>
<tr>
<td>ltv</td>
<td>The average level of Korean Mortgage Market</td>
<td>50%</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Technology sustainability: Assumption</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Together with [Table 3], I used the Metropolis-Hastings algorithm to estimate the actual key parameters. Estimated results are displayed in [Table 4] and [Figure 5], in which prior distribution and post estimation show the estimation results. The distribution function is adopted according to the Dynare approach\(^\text{14}\).

\(^\text{14}\) The estimation is based on specific bounds and priors as necessary. Dynare provides a variety of distribution functions for estimation. In particular, the distribution function can be used alternatively, such as beta, gamma, and normal distribution. The experiment for alternate function shows that the results are not significantly affected.
<Table 4> Prior and Posterior Distribution of Estimated Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean   Std. Err</td>
<td>Mean</td>
<td>Confidence</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99    0.01</td>
<td>0.9895</td>
<td>0.9904</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.10    0.50</td>
<td>2.1170</td>
<td>2.1247</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.40    0.07</td>
<td>0.3938</td>
<td>0.3940</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.21    0.07</td>
<td>0.2046</td>
<td>0.2047</td>
</tr>
<tr>
<td>$\omega_A$</td>
<td>1.00    0.01</td>
<td>0.9995</td>
<td>0.9822</td>
</tr>
</tbody>
</table>

According to the result, the EIS is 0.39, implying that household service and non-durable goods consumption are strong complements to each other. The implied value of the IES from 0.21 is 0.49. The estimate of the EIS is smaller than that of the IES, 0.49, which supports Hong (2010)'s assumption that the smaller EIS estimate than the IES explains the equity premium puzzle, as in Piazzesi, Schneider, and Tuzel (2006). [Figure 5] shows the prior and post estimation for the EIS parameter, $\epsilon$. [Figure 6] shows the prior and post estimation for subjective discount factor, $\beta$, and the IES, $\sigma$.

![Figure 5: The EIS Prior(Distribution) and Post(Right) Estimation](image-url)
5. Correlation experiment among consumption, housing, and housing collateral

I conduct a correlation experiment for different levels of the EIS with respect to housing, consumption, and collateral based on the Bayesian simulations. The level of the EIS determines the extent that housing is correlated with consumption. A smaller EIS can be translated into a higher correlation between housing and consumption. This experiment implies that as housing strongly complements consumption, the comovement between the two series can be strongly generated. Hence, the presence of complementarity can help

<table>
<thead>
<tr>
<th>Table 5: Implication of different value of the EIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>EIS = 0.39</td>
</tr>
<tr>
<td>correlation</td>
</tr>
<tr>
<td>consumption</td>
</tr>
<tr>
<td>housing</td>
</tr>
<tr>
<td>collateral</td>
</tr>
<tr>
<td>EIS = 0.5</td>
</tr>
<tr>
<td>correlation</td>
</tr>
<tr>
<td>consumption</td>
</tr>
<tr>
<td>housing</td>
</tr>
<tr>
<td>collateral</td>
</tr>
<tr>
<td>EIS = 0.9</td>
</tr>
<tr>
<td>correlation</td>
</tr>
<tr>
<td>consumption</td>
</tr>
<tr>
<td>housing</td>
</tr>
<tr>
<td>collateral</td>
</tr>
</tbody>
</table>
macroeconomic models with housing to produce significant comovements between macroeconomic variables and housing. [Table 5] presents correlation simulation experiments among housing, consumption, and collateral facing different values of the EIS based on the Bayesian simulations.

V. Robustness Check : CCR (Canonical Cointegration Regression) Estimation

1. Empirical model for CCR Estimation

I check the robustness of the estimation result as in [Table 3] through a CCR method, using Equation (1) through Equation (8). The model structure for CCR estimation takes Equation (1) and (5) with the price of non-durable goods as $p$, and the price of housing service as $h$. The use of the relative price is applied in Equation (3). The procedures and approaches are borrowed from Song(2012). To apply the co-integration test, each time-series of the consumption of non-durable goods and housing service and the prices of consumer goods and housing service should be a difference-stationary series. The parameter to be estimated from the right side of Equation (12) is the EIS, $\varepsilon$.

2. Estimation from Canonical Cointegration Regression (CCR)

It should be noted that the coefficient of difference-stationary time series does not follow a normal $t$-distribution in the estimation of the EIS. To implement the CCR method, the following is defined: $y_t = \log(C_t/D_t)$ and $x_t = \log(h_t/p_t)$. The estimation through the CCR is conducted to remove the long-term dependency between the cointegrating equation and stochastic movement, then followed by the estimation of covariance parameter. The model for CCR analysis is expressed as below\textsuperscript{15}. 

\[ y_t = c'x_t + \varepsilon_{f,t} \]  

(13)

In this formula, \( \nabla x_t = v_t, w_t = (\varepsilon_{f,t}, v_t)' \).

\[ \Phi(i) = E(w_i w_{t-i}'), \sum = \Phi(0), \Gamma = \sum_{i=0}^{\infty} \Phi(i), \] and \( \Omega = \sum_{i=-\infty}^{\infty} \Phi(i) \).

\[
\begin{align*}
y_t^* &= y_t + \pi_y^* w_t \\
x_t^* &= x_t + \pi_x^* w_t
\end{align*}
\]  

(14) (15)

Both \( y_t^* \) and \( x_t^* \) are cointegrated, because they are stationarized as a cointegration coefficient of \( \left( \begin{array}{c} 1 \\ -c \end{array} \right) \). [Table 6] displays the CCR estimation result. The dependent variable is \( \log(C/D) \) with sample periods from 1991 Q1 to 2011 Q1. This result shows the coefficient of the relative price is 0.78, which is corresponding to previous Bayesian estimation results, since the EIS is significantly lower then 1. It can be said that housing complements consumption.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Price</td>
<td>0.78</td>
<td>0.17</td>
<td>4.53</td>
<td>0.00</td>
</tr>
</tbody>
</table>

VI. Implications of Results and Limits

As the complementary relationship between housing and consumption emerges strongly, both housing demand and consumption would like to strongly rise. In the case when housing and consumption are in no complementary relationship and are instead engaged in a strong substitution relationship, housing demand would show an upward trend, but savings to purchase a house would also rise at the expense of consumption (thus consumption would decrease). With a stronger substitution relationship between housing and consumption,

15) Ogaki(2007) used the CCR method for cointegrating estimation, which is adopted in this chapter.
changes in housing would develop in the opposite direction of consumption, likely diminishing the impacts on the overall economic fluctuations. To put it another way, with expectations for an increase in housing prices, strong substitution between housing and consumption would cause a rise in housing demand, but would lead to a decrease in consumption due to the substitution effect (Kim, 2010).

[Figure 7] shows the operation of the strong complement characteristics, which play a crucial role in explaining macroeconomic fluctuations such as GDP, inflation, and interest. The significance of the impact on macroeconomic variables may be to the extent that the EIS determines complementarity. Suppose there is an exogenous impact on the demand of housing. The strong complementary relationship causes a strong rise in both housing and consumption, which creates a strong wealth effect and leverage effect. The house price changes accordingly and the house under a mortgage loan causes the comovement between housing and consumption due to complementarity. This scenario is also consistent with the conclusion made by Kim (2010).

![Diagram](source: Song(2012))

<Figure 7> Complementarity between Housing and Consumption
Meanwhile, this paper has some limitations as in the following: 1) this paper aims to mainly see the characteristics of housing and consumption through the estimation of the EIS. Since the empirical model is derived from the partial equilibrium mode, the effect of the EIS on macroeconomic variables may need to be richly analyzed in the general equilibrium model. In particular, the study on firms, which may use real estate as collateral for production financing, may be constructed to see a financial accelerator mechanism in the future research. 2) In order to calculate Korean average LTV, different levels of housing collateral should be weighed with respect to different LTV constitutions and distributions by households. In the case of some households, houses are not used as collateral at all. In the future study, the counter-factual study of different LTVs on macroeconomic variables would be conducted.

VII. Conclusion

In the past, housing was merely one of several types of assets and therefore the change in housing prices, like that in stock prices, was not considered as a major driving force that affects the real economy. This benign ignorance has been challenged later by a growing consensus on huge economic and social costs after having experienced considerable economic setback since the financial crisis in 2008. In particular, Kiyotaki and Moore (1997) argue that housing collateral as a financial accelerator plays an important role in a faithful manner of generating economic fluctuations.

The Bayesian estimation of the EIS in macro-data shows the complementary relationship between housing and consumption. This finding helps macroeconomics models to cause significant impacts on house price-driven economic fluctuations. According to the results, the Bayesian estimation method based on the housing collateral empirical model provides the EIS estimate of 0.39, indicating that both interact with each other as strong complements. Furthermore, to test the robustness of the Bayesian result, the CCR method is conducted for
the EIS estimation. Its result is consistent with the Bayesian approach.

A matter to note in the results is that housing and consumption are complements to each other. These results infer that the non-separability over housing and consumption in the macroeconomic housing models generate stronger comovements between the two series. The mutual interactions between the operation of EIS and IES would richly help analyze the effect of policy changes on macroeconomic fluctuations. Hence, such a result will help construct housing macroeconomic models such as DSGE models that better fit the comovement between housing and consumption\(^{16}\), showing complementarity. The existence of a strong complementary relationship makes housing an accelerating engine in business cycles. In addition, the mutual interactions between the EIS and IES can play an important role in data-fitting in the models. Moreover the models can provide a rich analysis about the effects of policy changes on economic variables. Subsequent studies may need to theoretically and empirically define and measure the impact degree of the complementary relationship on economic fluctuations.

References

4. Flavin, M. and Nakagawa, “A Model of housing in the presence of adjustment costs: A

\(^{16}\) To define the relationship between housing and non-housing consumption may be a bit controversial, since the non-housing consumption has the mixed nature of durable and non-durable goods, but nonetheless, since a few proceeding literatures made attempts to define the non-housing consumption, it would be meaningful to empirically analyze characteristics of these two goods in future studies.


국문요약

주택과 소비 : 기간내대체탄력성을 중심으로

글로벌 금융위기 이후 주택과 소비간의 관계를 규명하는 분석은 그 어느 때 보다 활발히 이루어지고 있다. 특히, 주택이 가지는 고유한 특성을 이해하고 이러한 특성을 거시경제모형에 구축하여 거시경제와의 연계성을 이해하려는 시도가 활발하다. 그러나 주택과 소비 간 상호연관성이 거시경제에 미치는 효과 분석에 대해서는 아직 연구가 충분치 못하다. 본 연구는 주택과 거시경제 상호간 역할과 연계성 분석의 기초라 할 수 있는 주택의 고유한 특성을 규명하는데 중점을 두었다. 주택과 소비 간 역할을 규명하기 위한 작업은 주택과 소비 간 기간내대체탄력성 추정을 통해 이루어진다. 과거의 연구가 미사티어를 통한 공간적 추정 방법을 사용하여 주택과 소비가 상호 보완체의 역할을 하고 있음을 설명해주고 있으나 본 연구에서 나타난 거시테이터를 통한 베이지어 추정 방식은 새로운 시도라 할 것이다. 또한 이 이아가, 추정모형에 주택담보대출을 고려하여 현실을 새로이 반영하고 동시에 강건성 점검을 실시하였다. 먼저, 추정을 위한 실증모형 구축을 위해 주택담보를 통한 대표적 가계의 효용극대화 문제를 설정하고 1개 최적화 조건을 적용한 다음 오일러식을 도출하였다. 마지막으로 오일러식을 도구 선형화하여 베이지어 추정 모형을 구축하였다. 추정결과 주택과 소비 간 기간내대체탄력성은 0.39로 두 시계열간 상호 강한 보완성을 가지고 있음을 보여주었다. 결과의 강건성 점검을 위해 베이지어 실증모형을 그대로 적용하여 CCR(Canonical Cointegration Regression) 모형을 도출하였고 동일한 거시 데이터를 사용하여 기간내대체탄력성을 추정하였다. 그 결과 기간내대체탄력성은 0.78로 추정되어 상호 보완성을 나타내고 있음을 제시하였다 기간내 대체 탄력성의 수치가 작을수록, 즉 상호 보완성이 강할수록 주택과 소비는 강한 상관관계를 형성하고 있음을 베이지어 모형 시뮬레이션으로 파악하였다.

이와 같은 추정결과는 주택가격 상승 및 하락으로 인한 주택서비스 및 소비의 공조현상을 설명하기위한 거시경제모형을 구축하는 데 도움을 준다. 2008년 이후의 주택가격 경제모형은 이러한 보완성이 나타나는 균형모형을 대부분 고려하지 않았다. 또한 본 결과는 주택시장의 변화가 소비 및 다른 거시경제 변수에 어느 정도로 영향을 주는지